AC Stark effect

Note Title 10/10/2013

"Dipole potential" or "light shift"

Consider the Schrodinger equation for classical light interacting with a quantum atom:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H_I'|\psi\rangle \qquad |\psi\rangle = c_g|g\rangle + c_e|e\rangle$$

$$\begin{cases} i\hbar\dot{c}_g = \hbar\frac{\Omega}{2}e^{i\delta t}c_e \\ i\hbar\dot{c}_e = \hbar\frac{\Omega}{2}e^{-i\delta t}c_g \end{cases}$$ (ignoring the phase of the electric field)

<u>using</u>: $\tilde{c}_g = c_g e^{-i\delta t/2} \qquad \tilde{c}_e = c_e e^{i\delta t/2}$

the equation for $\dot{c}_e$ becomes:

$$i\hbar\frac{\partial}{\partial t}\left(e^{-i\delta t/2}\tilde{c}_e\right) = \frac{\hbar\Omega}{2}e^{-i\delta t/2}\tilde{c}_g$$

$$i\hbar\dot{\tilde{c}}_e e^{-i\delta t/2} + \frac{\delta}{2}\hbar\tilde{c}_e e^{-i\delta t/2} = \frac{\hbar\Omega}{2}e^{-i\delta t/2}\tilde{c}_g$$

similarly: $$\begin{cases} i\hbar\dot{\tilde{c}}_e = \hbar\frac{\Omega}{2}\tilde{c}_g - \frac{\delta}{2}\hbar\tilde{c}_e \\ i\hbar\dot{\tilde{c}}_g = \hbar\frac{\Omega}{2}\tilde{c}_e + \frac{\delta}{2}\hbar\tilde{c}_g \end{cases}$$

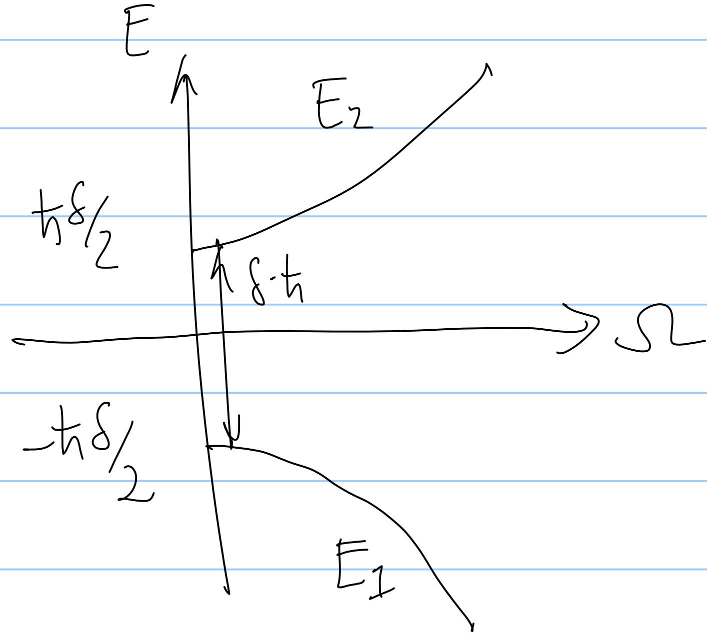
As for any quantum 2-level problem, these equations look like time evolution according to an effective Hamiltonian:

$$\tilde{H} = \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}$$

The energy of our "dressed states" $|1\rangle$ and $|2\rangle$ that diagonalize $\tilde{H}$ are:

$$E_1 = -\frac{\hbar}{2}\sqrt{\delta^2+\Omega^2} \qquad E_2 = \frac{\hbar}{2}\sqrt{\delta^2+\Omega^2}$$

In the far-detuned limit ($\delta \gg \Omega$), there is no significant excitation, but there are level shifts:

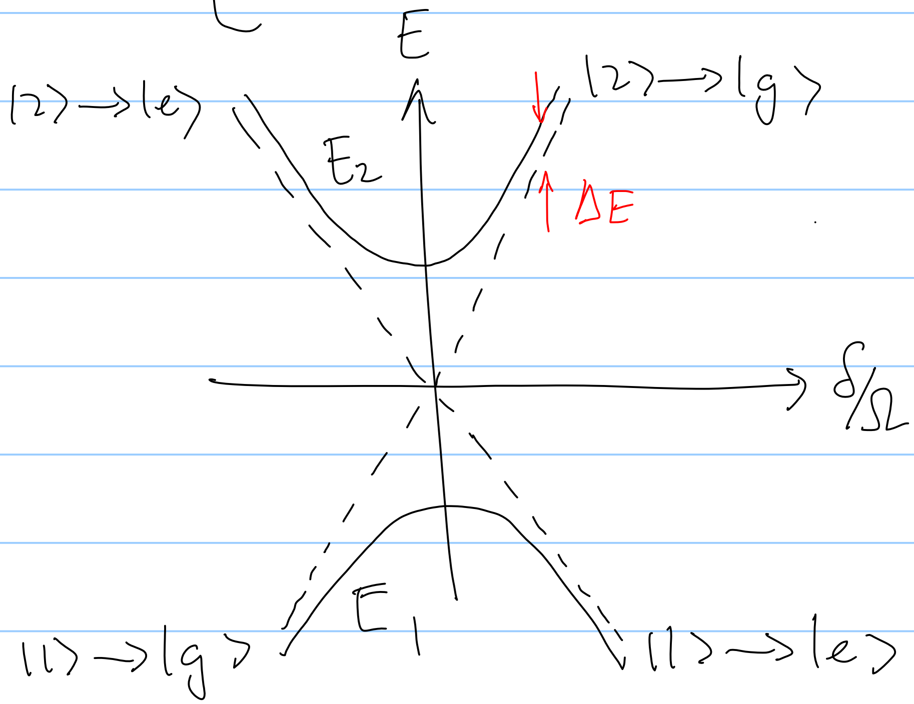


How do we identify the states with $|g\rangle$ and $|e\rangle$?

The eigenvectors of $\tilde{H}$ are:

$$|1\rangle \propto \left[\left(\frac{\delta}{\Omega} - \sqrt{1 + \left(\delta/\Omega\right)^2} \right) |g\rangle + |e\rangle \right]$$

$$|2\rangle \propto \left[\left(\frac{\delta}{\Omega} + \sqrt{1 + \left(\delta/\Omega\right)^2} \right) |g\rangle + |e\rangle \right]$$



Let's now find the energy shift ΔE from the interaction with the light for the ground state ($|2\rangle$ for $\frac{\delta}{\Omega} > 0$ and $|1\rangle$ for $\delta/\Omega < 0$).

@ $\Omega = 0$, $E_2 = \frac{\hbar\delta}{2}$ and $E_1 = \frac{-\hbar\delta}{2}$

$$\Delta E_2 = \frac{\hbar}{2}\sqrt{\delta^2+\Omega^2} - \frac{\hbar\delta}{2}$$

$$= \frac{\hbar}{2}\delta\left[\sqrt{1+\Omega^2/\delta^2} - 1\right]$$

$$\approx \frac{\hbar}{2}\delta\left[1+\frac{1}{2}\frac{\Omega^2}{\delta^2} - 1\right] \quad \text{for } |\Omega| \ll |\delta| \text{ or } \left|\Omega/\delta\right| \ll 1$$

$$= \frac{\hbar\Omega^2}{4\Omega}$$

Similarly:

$$\Delta E_1 = -\frac{\hbar}{2}\sqrt{\delta^2+\Omega^2} + \frac{\hbar\delta}{2}$$

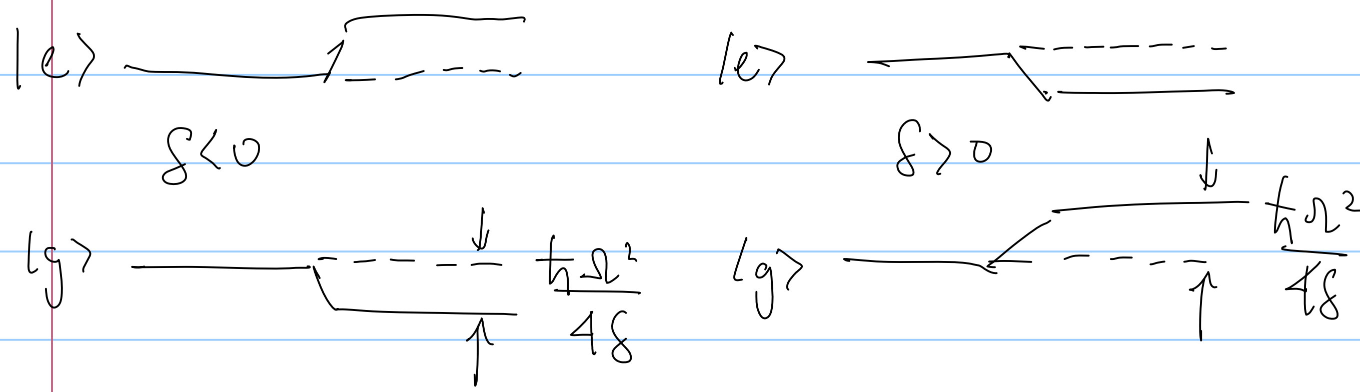
$$= \frac{\hbar}{2}\left(\delta - |\delta|\sqrt{1+\Omega^2/\delta^2}\right)$$

$$\approx \frac{\hbar}{2}\left(\delta - |\delta| - \tfrac{1}{2}\,\Omega^2/\delta^2\,|\delta|\right) \quad \text{for } \left|\Omega/\delta\right| \ll 1$$

$$= \frac{-\hbar\Omega^2}{4|\delta|} = \frac{\hbar\Omega^2}{4\delta}$$

(since $\delta < 0$ when $|1\rangle$ is the ground state)

Here's what the light does:



The same analysis for the excited state shows that it shifts by the same amount but in the opposite direction.